BOOK REVIEWS

A. V. Lykov

THE THEORY OF HEAT CONDUCTION*

Reviewed by A. G. Temkin

The fundamental significance of the analytical theory of heat conduction for the entire study of heat and mass transfer lies in the fact that it provides mathematical methods for the solution of problems dealing with the transport of energy and matter. Its general scientific value was highly regarded by Engels who referred to Fourier's classical work as a "mathematical poem," which became the basis of and the example for numerous research projects in the most varied branches of natural science and technology.

The effect of the analytical approach has been noticeably strengthened in recent times as a result of the development of analog techniques and computers. It is difficult to conceive of a more convenient and clear methodological approach for the training of students in the setting up of differential equations and boundary conditions or conditions of singularity, corresponding to the most diverse technological processes of modern industry. This refers not only to the traditional heat-engineering problems of energy-conversion machinery, construction, the manufacture of chemicals and cryogenics, rocket and atomic engineering, but also to industries concerned with organic synthesis, medicine, and biophysics, where the equations of transport and conservation offer rather exact descriptions of the concentrations of bacteria, viruses, and medicines in their convective and spontaneous motion, multiplication, and destruction.

It is for these reasons that the new ideas from the theory of heat conduction – as outlined in the latest book by Lykov – assume significant and fundamental importance for all related sciences and are deserving of attentive study.

First of all, we should note the significant updating of the mathematical apparatus, as compared to the standard courses in operational calculus.

Of particularly outstanding significance for the solution of complex heat-conduction problems is the theorem relating to the asymptotic expansion of an original, which is found from the exact expression for the transform. This Lykov theorem makes it always possible to find the principal part of a temperature distribution when the effects of the initial conditions have died away. Thus the field of action can be found even if the temperature of the medium or the heat flow is not an analytic function of time and has a finite number of discontinuities. The importance of this new theorem becomes clear if we note that Poisson's formula gives the asymptotic expansions even for small times, when the effects of the initial distribution are still important and when the input does not entirely govern the process. Jacobi's theta-functions are used in a convincing demonstration of the method. To prove the basic theorem and its consequences, the author constructed an extremely compact outline of the theory of analytical functions, developing his generalization of the Heaviside theorem for the case of multiple roots of an image denominator.

The text cites an inversion formula which does not require calculation of the Mellin–Riemann contour integral. With this formula we can derive the original of the function $f(\tau)$ from its image $F(s)$ by passing to the limit on the real straight line (pp. 52, 506):

$$f(\tau)=\lim_{n\to\infty}\left[\frac{(-1)^n}{n!}\left(\frac{n}{\tau}\right)^{n+1}F^{(n)}\left(\frac{n}{\tau}\right)\right].$$

*A text for students of heat-engineering sciences in higher educational institutions of the USSR, Vysshaya Shkola, 1967.

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Thus the solution of numerous heat-conduction problems is approached from another level, because the examination of the entire problem is possible while remaining at the real time axis, without turning to the complex plane of the operational argument. Lykov illustrates the capacity of this inversion formula with simple and convincing examples.

New concepts have been developed by Lykov in studying the conduction transfer of heat. Since the internal energy gradient u or the enthalpy gradient i (J/cm^3) are proportional to the corresponding volume heat capacity c_V or to c_p and to the temperature gradient,

$$\frac{\nabla u}{c_v} = \frac{\nabla i}{c_p} = \nabla t,$$

the heat flow q can be presented as the product of the corresponding thermal diffusivity a_V or a_p and the gradient of one of the functions of state:

$$\bar{q} = -a_v \nabla u = -a_p \nabla i.$$

In this modification of the Fourier law, the coefficient of the thermal diffusivity is the diffusion coefficient for the internal energy or the enthalpy as a function of the external conditions of the process. The conduction transfer of heat turns essentially into a diffusion process and the analogy between the equations of heat conduction and diffusion becomes clear in terms of its physical content. These equations describe the shifting of energy or matter which arises as a result of the concentration gradient. They express the process of leveling off the concentrations of the functions of state.

The time of thermal relaxation for matter is introduced into the literature for the first time. For macroscopic theory, the mechanical analogies are more natural than the resort to the kinetic Boltzmann equation. The author leads the reader to an examination of relaxation phenomena through the analogy of the shearing stresses of a rheological medium. It is precisely this approach which is appropriate in this case; it is easily associated with the hydrodynamic theory of heat transfer that had been developed by Reynolds. At no time had any relationship been expressed in the literature between the relaxation phenomena and the kinematics of a temperature field, which is based on the calculation of the velocity of motion for an isothermal surface. This velocity, equal to the product of the logarithmic derivative of the energy flux and the thermal diffusivity a_p, has never before been associated with the speed of sound. Such an examination is quite timely. In dilatometric methods of determining the thermophysical parameters of transport, the isotherms are determined experimentally.

This solid mathematical apparatus enables us to deal with the most complex problems of heat conduction at a very contemporary level. In conjunction with the classical method of separating variables, the author considers in rather considerable detail the numerical methods of solution, as well as integral transformations. Lykov demonstrates the general principle of their construction, the selection of the kernel, and the integration domains. In addition to the Laplace transformation, the various transforms of Fourier and Hankel are shown fully and meaningfully. The appropriate inversion formulas and tables of transforms are presented for these. Of particular methodological value is the fact that the book compares the methods of solution. The separation of variables is compared with the Laplace and Fourier transforms by the method of sources. All of these solutions are beautifully nomographed and the graphs will doubtlessly find their way into the handbooks. In particular, this has been done for the fields in which the temperature of the medium varies linearly, exponentially, and harmonically. For temperature waves within a semiinfinite body, sphere, cylinder, and plate all of the necessary tables of amplitude and phase values are given. The relationship between the amplitude of the surface temperature and the Biot number is demonstrated, as is the function relating the coefficient of heat accumulation and heat utilization to the Predvoditelev number. Prior to the Lykov book these quantities had not been described in the literature.

The temperature fields of exothermic bodies in the case of variable external effects has been treated in the most general form with the Duhamel integral. The Borel convolution is used to deal with the field of the source, which depends in a general way on the coordinates and time. The isolation of the principal part of the solution makes it possible, in the case of a constant temperature for the medium, for the heat flow, and for a stable internal heat source, to generalize the regular regime which is common to all fields of this type.

The main part of the book is devoted to an examination of the heat-conduction processes under boundary conditions of the first, second, and third kind, i.e., when the temperature, its gradient, or the linear combination of these are specified at the surface of a body. In addition to these, the temperature fields for total contact between two bodies are studied systematically for the first time in the literature; here we are dealing with an equality of temperatures and heat flows on a surface common to two media. According to the Lykov classification, such contact between temperature fields is a boundary condition of the fourth kind. This special treatment of a singular boundary condition is needed because of its importance in the solution of all problems relating to convection heat transfer. It is not only – or more exactly, not so much – the solids which satisfy the boundary condition of the fourth kind (since between these there arises the resistance of the contact itself), but rather a solid and a liquid or a gas, given moderate and elevated pressures. In rigorous formulation, all problems of convection transport of energy and matter are stated in terms of this condition and all subsequent development of this important branch of science is impossible without solution of problems including the Lykov boundary condition. The solution of the problem dealing with the "transfer of heat between a body and the fluid flow streamlining that body" is an example of precisely this approach. The description "of the temperature field with a change in the aggregate state," in which the Leibenzon solution is refined, as well as the description of the processes "of heat conduction with variable transport coefficients," in which the methods of solving even nonlinear solutions are demonstrated with the greatest possible thoroughness, are associated with this main purpose of the book.

The Lykov text is remarkable for its depth of pedagogical thought. It is read easily and pleasantly, it is convenient for independent work, it leads the reader through the laboratory of the author's thoughts, into the range of contemporary scientific problems, and each thoughtful student will find in this book not only methods for the solutions of problems, but a powerful stimulus for independent research and creativity.

A. I. Pekhovich and V. M. Zhidkikh

CALCULATIONS OF THE THERMAL REGIME IN SOLIDS*

Reviewed by Sh. N. Plyat

The literature on the calculation of heat-transfer processes is currently quite extensive. However, we cannot help but take note of a significant gap – there is no book which contains even a summary of practical methods and theoretical graphs with which – without considerable expenditure of time – we can calculate nonsteady thermal processes.

It is precisely the fundamental purpose of this book to fill that gap, i.e., to provide engineers with a convenient and simple apparatus for the accomplishment of thermal calculations for solids.

The book consists of two parts – a methodological part and a part containing theoretical graphs.

The theoretical-graph part of the book contains materials which permit rapid determination of the temperature, the temperature gradient, and the enthalpy in bodies of varying geometric shape, given a variety of initial and boundary conditions, in the presence of internal (both instantaneous and continuous) and moving heat sources.

For this purpose the book contains more than 200 theoretical graphs, covering 59 problems of a nonsteady thermal regime in solids. There are graphs with which to determine the time for the onset of a regular regime. Moreover, additional data have been collected: simplified analytical functions, values for the coordinates and times of temperature and temperature-gradient peaks, etc. Each problem is accompanied by an example of the calculation. This portion of the book is of great practical interest, since it not only facilitates the accomplishment of thermal calculations, but it makes it possible to analyze the progress of the thermal process, to solve the inverse problems, etc.

The value of this portion of the work increases considerably, if we treat it in conjunction with the methodological portion of the book.

The sequence of problem solution is covered in the methodological portion of the book. Here it is shown how, on the basis of the physical essence of a phenomenon, to formulate a problem properly, and primarily how to designate the initial and boundary conditions. Here the concept of two types of heat sources is introduced – sources with a specified temperature and sources with a specified power.

If the problem under consideration coincides in terms of singularity conditions with any of the 59 problems for which the theoretical graphs have been plotted, one needs only make use of these graphs. However, the basic content of the methodological section of the book involves the demonstration of how to solve a multiplicity of similar extremely complex problems with the existing solutions for simple problems.

For this the principle of superposition is employed. The book provides a detailed and quite accessible demonstration of how a problem with complex initial and boundary conditions, and with thermophysical material characteristics variable in time can be broken down into the algebraic sum of simple problems whose solutions are known and for which, in addition, theoretical graphs have been plotted. As a result, the calculations are reduced to the simplest arithmetic operations.

The practical purpose of the book did not prevent the authors from treating a number of theoretical problems relating to nonsteady heat conduction. Among these problems are included primarily the analysis of possible applications of the method of superposition to the solution of thermal problems.

*Énergiya, 1968.

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In addition, it is our opinion that the idea of seeking a relationship between externally diverse phenomena which in actuality have much in common with each other is particularly useful. As a result we find the possibility of a more thorough exposition of the physical essence of the phenomena. Thus, for example, we find indications of similarity for the temperature regime of a semibounded body for boundary conditions of the Ist and IIIrd kinds, as well as for boundary conditions of the IInd kind, when there is a layer of a thoroughly turbulent liquid on the surface of the body. The analytical relationship determined for these conditions substantially simplifies the accomplishment of the calculations. Also useful are the relationships between the solutions of the problems and the exact, linear, and plane heat sources; in addition, we have the relationships between the problems with internal sources and the sources situated at the body surface.

It should be stressed that the material of the book can be applied not only to problems of heat propagation, but to all the phenomena which are described by the Fourier equation, i.e., the diffusion of matter, etc.

The combination of practical calculation procedures with the original theoretical analysis of thermal processes, the clarity, and the arrangement of the material make the Pekhovich and Zhidkikh book extremely interesting and useful for a wide range of scientists and engineers.